Genetic relationships and close inbreeding - solutions

1. a)

\[ F_A = \frac{1}{2} a_{BD} = \frac{1}{2} \times \left( \frac{1}{2} \right)^{1+0} \left( 1 + F_D \right) = \frac{1}{2} \times \left( \frac{1}{2} \times (1 + 0) \right) = \frac{1}{4} = 0.25 \]

b) \[ a_{AD} = \left( \frac{1}{2} \right)^{2+0} + \left( \frac{1}{2} \right)^{1+0} \left( 1 + F_D \right) = \left( \frac{1}{2} \right)^{2} + \left( \frac{1}{2} \right)^{1} (1 + 0) = \frac{1}{4} + \frac{1}{2} = 0.75 \]

2. a)

\[ a_{DS} = \left( \frac{1}{2} \right)^{1+2} \left( 1 + F_0 \right) = \left( \frac{1}{2} \right)^{3} = \frac{1}{8} \]

b) \[ F_{\text{progeny}} = \frac{1}{2} a_{DS} = \frac{1}{2} \times \frac{1}{8} = \frac{1}{16} = 0.625 = 6.25\% \]
3. a) \( F_Z = \) The probability that two alleles in a locus of an individual Z are identical by descent (IBD)

\[ Z \quad \text{Pr (IBD)} \]

b) \( f_{xy} = \) The probability that a randomly sampled gene from individual X is identical by descent with one randomly sampled gene of the same locus from individual Y

\[ X \quad \text{Pr (IBD)} \]
\[ Y \quad \text{Pr (IBD)} \]

\[ f_{xy} = \frac{1}{2} a_{xy} \]

c) \( a_{xy} = \) The probability that a randomly sampled gene from individual X is identical by descent with any of the genes in the same locus from individual Y

\[ X \quad \text{Pr (IBD)} \]
\[ Y \quad \text{Pr (IBD)} \]

\[ F_Z = f_{xy} = \frac{1}{2} a_{xy} \]

d) \[ X \quad Y \quad Z \]

4. D is inbred: \( F_D = \frac{1}{2} a_{BA} = \frac{1}{4} \)

Three paths must be summarized:

\[ a_{EF} = \frac{1}{2} a_{BA} = \frac{1}{4} \]

\[ = \left( \frac{1}{2} \right)^5 + \left( \frac{1}{2} \right)^4 + \left( \frac{1}{2} \right)^3 \times \frac{5}{2} + \frac{1}{32} + \frac{5}{16} = \frac{13}{32} \]

\[ F_X = \frac{1}{2} a_{EF} = \frac{1}{2} \times \frac{13}{32} = \frac{13}{64} \approx 0.20 \]
5.

a) 2069 M (♂)

The inbreeding coefficient of the progeny ($F_X$) equals half the additive relationship ($a_{AB}$) between parents A and B

$$a_{AB} = \left(\frac{1}{2}\right)^{m+n} \left(1 + F_{2069M}\right)$$

where $m$ and $n$ are the number of arrows between A, B and the common ancestor (2069 M)

Thus

$$a_{AB} = \frac{1}{2} (1+0) = \frac{1}{16} = 0.0625 \text{ or } 6.25\%.$$ 

The inbreeding coefficient for X is

$$F_X = \frac{1}{2} a_{AB} = \frac{1}{2} \times \frac{1}{16} \times \frac{1}{32} = 0.03125 \text{ or } 3.125\%.$$ 

Here we assumed that the ancestor (2069 M) is not inbred. If e.g. the inbreeding coefficient for 2069 M is 5%, then

$$a_{AB} = \left(\frac{1}{2}\right)^{2+2} (1+0.05) = 0.0625 \times 1.05 = 0.066 \text{ or } 6.6\% \text{ and}$$

$$F_X = 3.3\%$$

b) 2069 M

The relationship between the parents of Y is

$$a_{XF} = \left(\frac{1}{2}\right)^{3+2} + \left(\frac{1}{2}\right)^{3+2} \times \left(1 + F_{2069M}\right)$$

i.e. both paths X, A, C, 2069 M, E, F and X, B, D, 2069 M, E, F are summarized

If $F_{2069M}$ is 0 (2069 M not inbred)

$$a_{XF} = \frac{1}{32} + \frac{1}{32} = \frac{1}{16} = 0.0625 = 6.25\%$$

The inbreeding coefficient for Y is then

$$F_Y = \frac{1}{2} \times \frac{1}{16} = \frac{1}{32} = 0.03125 = 3.125\%$$
c) The heifer X is a grand-daughter to 1138 S and furthermore related to him through 2069 M. The relationship between them is

\[ a_{X-1138S} = (\frac{1}{2})^{2+0}(1+F_{1138S}) + (\frac{1}{2})^{3+1}(1+F_{2069M}). \]

If 1138 S and 2069 M are not inbred, i.e. \( F_{1138S} \) and \( F_{2069M} = 0 \) then

\[ a_{X-1138S} = (\frac{1}{2})^{2+0} + (\frac{1}{2})^{3+1} = \frac{1}{4} + \frac{1}{16} = \frac{5}{16} = 0.3125 = 31.25\%. \]

The inbreeding coefficient for an offspring, if the heifer X is inseminated by 1138 S would be:

\[ F_Y = \frac{1}{2} a_{X-1138S} = \frac{1}{2} \times \frac{5}{16} = \frac{5}{32} = 0.15625 = 15.6\%. \]

d) 15% is definitely too high to be recommended